Relations and functions

A relation is a set of ordered pairs. It is often defined by a certain equation. The domain of the relation is a set of the first elements of ordered pairs (the $x$-values). The range of the relation is the set of second elements of ordered pairs (the $y$-values). For example, for the relation \{(1, 2), (2, 5), (−1, 6), (4, −1)\},

\[
\text{Domain} = \{-1, 1, 2, 4\} \quad \text{and} \quad \text{Range} = \{-1, 2, 5, 6\}.
\]

Relations can be classified according to the type of correspondence as follows.

One-to-one correspondence occurs if for each value of $x$ there is one unique value of $y$. The above relation is an example of one-to-one correspondence.

Many-to-one correspondence occurs when there is more than one value of $x$ for the same value of $y$.

For example, the relation \{(-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)\} is an example of many-to-one correspondence, as there are two values of $x$ (−1, 1) for the same value of $y$ (1).

One-to-many correspondence occurs if for one value of $x$ there is more than one value of $y$. The relation \{(2, 1), (3, 7), (2, 5), (1, −2)\} is an example of one-to-many correspondence, since there are two values of $y$ (1 and 5) for the same value of $x$ (2).

Finally, there is many-to-many correspondence. This type of correspondence occurs when more than one value of $x$ maps onto more than one value of $y$. For example, \{(12, 3), (12, 2), (10, 5), (10, 2)\}.

A function is a type of relation such that there are no two ordered pairs with the same first element. That is, each element of the domain determines one and only one element of the range.
A function of \( x \) is denoted as \( f(x) \).

Hence, a function is a relation with a one-to-one or many-to-one correspondence. If the relation is represented graphically, there is a simple way to check whether the relation is a function or not. It is called the **vertical line test**. To apply the test we need to look at the graph of the relation and imagine (or draw) a series of vertical lines. If it is possible to draw a line which will intersect the graph in more than one place, the relation is not a function. If no vertical line will intersect the graph in more than one place, the relation is a function.

![Graph of a function and a non-function](image)

**3A Plotting straight line graphs**

A linear function is a set of ordered pairs that, when graphed, form a straight line. If the rule of the function is given, we can construct a table of values and plot the resultant ordered pairs on a set of \( x \)- and \( y \)-axes. 

*Note:* The rule of the function should be expressed with \( y \) as the subject to allow the table of values to be easily calculated.

**WORKED EXAMPLE 1**

Plot the graph of the linear equation \( y = 3x - 1 \) for values of \( x \) between \(-3\) and \(3\).

1. **THINK**
   - Construct a table of values for \(-3 \leq x \leq 3\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(-10)</td>
<td>(-7)</td>
<td>(-4)</td>
<td>(-1)</td>
<td>(2)</td>
<td>(5)</td>
<td>(8)</td>
</tr>
</tbody>
</table>

2. **WRITE/DRWA**
   - Plot the points and join them with the straight line.

![Graph of the linear equation](image)
WORKED EXAMPLE 2

Plot the graph of the linear equation \( y + 5x - 2 = 0 \).

**WRITE/DRAW**

\[
y + 5x - 2 = 0
\]

\[
\therefore y = -5x + 2
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>12</td>
<td>7</td>
<td>2</td>
<td>-3</td>
<td>-8</td>
</tr>
</tbody>
</table>

1. **THINK**
   - Rearrange the equation to make \( y \) the subject.
   - Construct a table of values.
   - Plot the points and join them with a straight line.

**REMEMBER**

1. A relation is a set of ordered pairs.
2. The domain of the relation is a set of the first elements of ordered pairs (\( x \)-values). The range — is a set of the second elements of ordered pairs (\( y \)-values).
3. A function is a type of relation where each element of the domain determines one and only one element of the range. A function of \( x \) is denoted as \( f(x) \).
4. To plot a straight line graph:
   - (a) transpose the equation to make \( y \) the subject
   - (b) construct the table of values
   - (c) plot the ordered pairs from the table and join them with a straight line.
5. If the domain is restricted, make sure the graph is not extended beyond the specified values.
6. If the domain is not restricted, arrows are sometimes placed on both ends to indicate that the line extends in both directions.

**EXERCISE 3A**

Plotting straight line graphs

1. **WRITE**
   - Plot the graphs of the following linear equations for values of \( x \) between 3 and 3.

   - \( a \quad y = 5x \)
   - \( c \quad y = -2x + 6 \)
   - \( e \quad y = \frac{1}{2} x + 2 \)
   - \( g \quad y = 4x - 3 \)
   - \( i \quad y = x - 3 \frac{1}{2} \)
   - \( b \quad y = x + 2 \)
   - \( d \quad y = -3x + 5 \)
   - \( f \quad y = x - 1 \)
   - \( h \quad y = 2x - 2 \)
   - \( j \quad y = 1 - 3x \)
2 **WE2** Plot the graphs of the following linear equations.

\[
\begin{align*}
\text{a} & : x = 2 - y \\
\text{b} & : 5 = x - y \\
\text{c} & : x - y - 6 = 0 \\
\text{d} & : -13 = x + y \\
\text{e} & : y - x = 7 \\
\text{f} & : y + x = 4 \\
\text{g} & : y + 2x - 3 = 0 \\
\text{h} & : y - 3x + 1 = 0 \\
\text{i} & : 2y - 3x + 4 = 0 \\
\text{j} & : 3y + 4x - 6 = 0
\end{align*}
\]

3 Based on your observations from question 2, predict which of the following equations represent linear functions. Give reasons for your answer.

\[
\begin{align*}
\text{a} & : y = x \\
\text{b} & : y = 8x - 11 \\
\text{c} & : y = -0.5x + 1.5 \\
\text{d} & : y = x^2 + 2 \\
\text{e} & : y = \frac{3}{x} \\
\text{f} & : y - x + 7 = 0 \\
\text{g} & : y - 3x = 0 \\
\text{h} & : y = xy - 1 \\
\text{i} & : x = 2y \\
\text{j} & : 3y = x + 2
\end{align*}
\]

4 **MC** One point on the line \( y = 2x + 3 \) has coordinates:

- A \((-2, 3)\)
- B \((2, 12)\)
- C \((-1, -1)\)
- D \((-2, -1)\)
- E \((1, 4)\)

5 **MC** The points \((3, -1)\) and \((-1, -3)\) lie on the line with equation:

- A \(x + y = 2\)
- B \(2x - y = 1\)
- C \(x - 2y = 2\)
- D \(2x + y = 5\)
- E \(x - 2y = 5\)

3B **Using a CAS calculator to plot and sketch linear functions**

A CAS calculator can be used to assist you in creating tables of values and plotting linear functions.

**WORKED EXAMPLE 3**

**Plot the graph of** \( y = x + 2 \) **using a CAS calculator.**

**THINK**

1. Create a table of values for \( x \) between \(-3\) and \(3\) inclusive.
   To do this, on the Spreadsheet screen:
   - Label column A: ‘\( x \) values’
   - Label column B: ‘\( y \) values’
   - Then press \( \text{OK} \).
   - Complete the entry line in cell B2 as:
     \( y = A2 + 2 \)
   - Then press \( \text{OK} \).
   - Highlight cells B2 to B8 and tap:
     - Edit
     - Fill Range
     - OK
To plot the graph, highlight the values and tap:
- Graph
- Scatter
To join the points with a straight line, tap:
- View
- Lines

A CAS calculator can also be used for sketching graphs of linear functions and finding coordinates of the axial intercepts.

**WORKED EXAMPLE 4**

Graph the function \( y - 5x + 2 = 0 \) using a CAS calculator. Then sketch the graph in your workbook, showing \( x \)- and \( y \)-intercepts.

**THINK**

1. Rearrange the equation to make \( y \) the subject.
2. On the Graph & Table screen, complete the entry line as:
   \[ y1 = 5x - 2 \]
   Tick the \( y1 \) box and tap \( \text{[F1]} \).
   To see the coordinates of the \( y \)-intercept, tap:
   - Analysis
   - G-Solve
   - \( y \)-intercept
   The coordinate will appear on the graph.

3. To find the \( x \)-intercept, tap:
   - Analysis
   - Root
   The coordinate will appear on the graph.
Sketch the graph in your workbook, showing the coordinates of $x$- and $y$-intercepts.

REMEMBER

1. If necessary, rearrange the equation to make $y$ the subject.
2. Always check whether the graphing window needs adjustment.

EXERCISE 3B

Using a CAS calculator to plot and sketch linear functions

1. **WE3** Plot the graphs of the following linear equations using a CAS calculator.
   - a) $y = 5x$
   - b) $y = x + 2$
   - c) $y = -2x + 6$

2. For the linear function $y = 0.1x - 0.341$, copy and complete the following table by using a CAS calculator. Give answers to 2 decimal places.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. **WE4** Graph the following linear functions using a CAS calculator. Then sketch the graphs in your workbook, showing $x$- and $y$-intercepts.
   - a) $y = -3x + 5$
   - b) $y = x - 7$
   - c) $y = x + 4$
   - d) $y + 2x - 3 = 0$
   - e) $y - 3x + 1 = 0$
   - f) $2y - 3x + 4 = 0$
   - g) $3y + 4x - 6 = 0$

4. Consider the linear equation $y = 0.1x + 0.08$.
   - a) Copy and complete the following table of values for $x$ between $-1$ and 1 by using a CAS calculator.
     | x  | -1 | -0.8 | -0.6 | -0.4 | -0.2 | 0  | 0.2 | 0.4 | 0.6 | 0.8 | 1  |
     | y  |    |      |      |      |      |    |     |     |     |     |    |
   - b) Hence, adjust the window settings to clearly show the graph of the function for $-1 \leq x \leq 1$ and sketch using a CAS calculator.
   - c) Find the $y$-intercept (to 2 decimal places), using a CAS calculator.
   - d) Find the $x$-intercept (to 1 decimal place), using a CAS calculator.
   - e) Sketch the linear graph for the given domain in your workbook, showing intercepts with the axis.
   - f) Find the value of $y$ when $x = 0.05$.

5. Consider the following linear equation: $y - x + 200 = 0$.
   - a) Rewrite the equation to make $y$ the subject.
   - b) Use a CAS calculator to create a table of values, starting at $x = -10$ with increments of 20. If $X_{\text{min}} = -10$ and $X_{\text{max}} = 210$, determine from the table the $Y_{\text{min}}$ and $Y_{\text{max}}$ values and adjust the window settings accordingly.
c Graph the function on the calculator.
d Determine the y-intercept.
e Sketch the graph showing the y-intercept and giving the coordinates of one other point.

6 Sketch the following linear functions on a CAS calculator by choosing appropriate window settings to show the graph intersecting both axes.

Sketch each graph, showing the coordinates of the x- and y-intercepts.

a \( y = 5x - 120 \)
b \( y = -6x + 154 \)
c \( y = -0.6x - 0.01 \)
d \( y = -1.25x - 0.125 \)
e \( 20y - 14x = 1000 \)
f \( 2y + 3x = 120 \)

### 3C Finding the gradient of a straight line

The slope of a line is called the gradient, \( m \). It can be calculated by finding the ratio of the rise and run between any two points on the line \((x_1, y_1)\) and \((x_2, y_2)\):

\[
m = \frac{\text{rise}}{\text{run}}
\]

Between the two points with coordinates \((x_1, y_1)\) and \((x_2, y_2)\), rise is calculated by finding the difference between the y-values \((y_2 - y_1)\) and run by finding the difference between the x-values \((x_2 - x_1)\).

Hence, the gradient (that is, the ratio of rise and run) is given by

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Note also that order is not really significant. However, we usually take the points from left to right as \((x_1, y_1)\) and \((x_2, y_2)\).

### WORKED EXAMPLE 5

Calculate the gradient of the line shown at right.

\[
\begin{align*}
& (2, 10) \\
& (-1, 4)
\end{align*}
\]

**THINK**

1. Select two points on the line and state their coordinates.
2. Calculate the gradient.

**WRITE**

Let the point \((-1, 4)\) be \((x_1, y_1)\) and the point \((2, 10)\) be \((x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{2 - (-1)} = \frac{6}{3} = 2
\]

Hence, the gradient of the line is 2.
Gradient is calculated by finding the ratio of the rise and run between any two points on the line $(x_1, y_1)$ and $(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**EXERCISE 3C** Finding the gradient of a straight line

1. Find the gradients of the following lines.
   
   a.
   
   
   b.
   
   c.
   
   d.
   
   e.
   
   f.
   
   g.
   
   h.
   
   i.
   
   j.

2. Find the gradients of the lines joining these points.
   
   a. (0, 0) and (2, 6)  
   b. (0, 4) and (2, 6)  
   c. (0, −6) and (8, 0)  
   d. (0, 3) and (−6, 0)  
   e. (−1, 5) and (1, 11)  
   f. (−3, 1) and (−1, 5)  
   g. (−2, 3) and (3, −7)  
   h. (−6, 2) and (2, 10)  
   i. (−1, −6) and (2, 7)  
   j. (−7, −2) and (−5, −8)

3. A line intersects the $y$-axis at $y = 2$ and the $x$-axis at $x = −3$. Find the gradient of the line.

4. A line has a gradient of 2 and passes through the point $A (1, 4)$ and the point $B (3, y)$. Find the value of $y$. 

**Digital docs**

**SkillSHEET 3.3**

**Gradient of a straight line**

**Spreadsheet 046**

**Gradient**

**Maths Quest 11 Standard General Mathematics for the Casio ClassPad**
5 (MC) A straight line intersects the x-axis at $x = -4$ and the y-axis at $y = -1$. Which of the following does not give the value of the gradient?

A \[ \frac{0 - (-1)}{-4 - 0} \]
B \[ \frac{-1 - 0}{0 - (-4)} \]
C \[ \frac{(-1) - 0}{(-4) - 0} \]
D \[ \frac{-1}{4} \]
E \[ -0.25 \]

6 (MC) A line which passes through the points M (2, -4) and N (a, -8) has a gradient of $\frac{1}{4}$. The value of $a$ is:

A 4  B 14  C 4  D 8  E 16

7 (MC) Examine the linear graph at right. Which of the following statements is not true?

A The line has a positive gradient.
B The y-intercept has coordinates (0, 3).
C The gradient of the line is smaller than 1.
D The slope of the line is equal to $\frac{-3}{5}$.
E The x-intercept has coordinates (5, 0).

8 In the illustration at right, the bottom of the playground slide is 2.5 m from the foot of the ladder. The gradient of the line which represents the slide is 0.68. How tall is the ladder?

9 Estimate the gradient of the ski slope by measuring the appropriate red lines.

10 Find the gradient of the hillside vineyard shown by taking suitable measurements between the points A and B.
Determining the equation of a straight line

The gradient–intercept form of the equation of a straight line is given by

\[ y = mx + c \]

where \( m \) is the gradient and \( c \) is the \( y \)-intercept.

Hence, to determine the equation of a straight line we need to establish the values of \( m \) and \( c \) first. The method of attacking problems depends on the information given in each particular question.

Finding the equation of a straight line, given its gradient and \( y \)-intercept

When both the gradient and the \( y \)-intercept are known, determining the equation of the line amounts to substitution of the given values into the equation \( y = mx + c \).

**WORKED EXAMPLE 6**

Find the equation of a line which has a gradient of 2 and \( y \)-intercept equal to 3.

**THINK**

1. Write the general equation of a straight line.
2. Identify the values of \( m \) and \( c \).
3. Substitute the given values for \( m \) and \( c \) into the equation.
4. Write the answer.

**WRITE**

1. \( y = mx + c \)
2. \( m = 2 \) and \( c = 3 \)
3. \( \therefore y = 2x + 3 \)
4. Hence, the equation of the line is \( y = 2x + 3 \).

Finding the equation of the straight line, given its gradient and one point on the line

Sometimes we are given the value of the gradient and the coordinates of a point which belongs to the line, but is not the point of intersection of the line with the \( y \)-axis. In a situation like this, we have to establish the value of \( c \) in order to be able to write the equation of the line.

**WORKED EXAMPLE 7**

Find the equation of the straight line with a gradient of 4 and passing through the point \((-5, 6)\),

\[ a \] by hand \hspace{1cm} \[ b \] using a CAS calculator.

**THINK**

\[ a \]

1. Write the general equation of a straight line.
2. Identify the value of \( m \) and substitute it into the equation.
3. Substitute the coordinates of the given point \((-5, 6)\) for \( x \) and \( y \) into the equation \( y = 4x + c \).

**WRITE**

\[ a \]

1. \( y = mx + c \)
2. Since \( m = 4 \)
   \[ y = 4x + c \]
\[ \text{Given the point } (-5, 6) \text{ lies on the line:} \]
   \[ 6 = (4)(-5) + c \]
Solve for $c$.

Substitute the value of $c$ into $y = 4x + c$ and write the answer.

On the Main screen, complete the entry line as:

\[
solve(y = mx + c, m = 4 | x = -5 | y = 6)
\]

Then press \( \text{Solve} \).

(1) Substitute the values of $m$ and $c$ into the equation $y = mx + c$ and write the answer.

\[
m = 4 \\
c = 26
\]

Hence, the equation of the line is $y = 4x + 26$.

---

**Finding the equation of the straight line, given two points on the line**

In some cases neither the $m$ nor $c$ values are known. The coordinates of two points on the line can be used to establish the value of $m$ (as was shown in the previous section). Once the gradient is found, the problem reduces to the one discussed above (that is, finding the equation, given the gradient and one point), except that instead of one point we have two to choose from.

**WORKED EXAMPLE 8**

Find the equation of the straight line passing through the points $(1, -2)$ and $(3, -7)$,

a by hand

b using a CAS calculator.

**THINK**

a 1 Write the general equation of a straight line.

2 Use the coordinates of the given points to find the value of the gradient.

**WRITE**

a $y = mx + c$

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Let the point $(1, -2)$ be $(x_1, y_1)$ and the point $(3, -7)$ be $(x_2, y_2)$.

\[
m = \frac{-7 - (-2)}{3 - 1} = \frac{-5}{2}
\]
3 Substitute the gradient and the coordinates of any of the two points (say, the first one) into the general equation to find $c$.

4 Solve for $c$.

5 Write the final equation. (Each term of the equation can be multiplied by 2 to get rid of fractions).

Using the point $(1, -2)$ and $m = -\frac{5}{2}$:

\[-2 = \left(-\frac{5}{2}\right)(1) + c\]
\[2 = -\frac{5}{2} + c\]
\[-\frac{1}{2} = c\]
\[\therefore c = \frac{1}{2}\]

Hence, the equation of the line is $y = -\frac{5}{2}x + \frac{1}{2}$ or $2y = -5x + 1$.

b 1 On the Statistics screen, label list1 as ‘xco’ and enter the $x$-coordinates of the two points.
Label list2 as ‘yco’ and enter the $y$-coordinates of the two points.

2 To plot the points, tap:
   • SetGraph
   • Setting
   Set:
   Type: Scatter
   XList: main\xco
   YList: main\yco
   Freq: 1
   Mark: Square

   • Set
   •

3 To determine the equation of the line joining $(1, -2)$ and $(3, -7)$, tap:
   • Calc
   • Linear Reg
   Set:
   XList: main\xco
   YList: main\yco
   Freq: 1
   Then tap OK.
Write the equation.  

The equation of the line is:  

\[ y = -2.5x + 0.5 \]  

Or  

\[ 2y = -5x + 1 \]

**REMEMBER**

1. The equation of a straight line is given by \[ y = mx + c \], where \( m \) is the gradient and \( c \) is the \( y \)-intercept.
2. Parallel lines have the same gradient.

**EXERCISE**

**Determining the equation of a straight line**

1. **WE6** Find the equation of a line which has:
   - \( a \) gradient 2, \( y \)-intercept 1
   - \( b \) gradient 3, \( y \)-intercept 4
   - \( c \) gradient \(-1\), \( y \)-intercept 5
   - \( d \) gradient \(-4\), \( y \)-intercept \(-2\)
   - \( e \) gradient \( \frac{1}{2} \), \( y \)-intercept \( 2\frac{1}{2} \)
   - \( f \) gradient \( \frac{2}{3} \), \( y \)-intercept \( -3\frac{1}{3} \)
   - \( g \) gradient \( \frac{3}{2} \), \( y \)-intercept \( -1\frac{1}{5} \)
   - \( h \) gradient \( -\frac{3}{7} \), \( y \)-intercept \( \frac{1}{2} \).

2. **WE7** Find the equations of the following straight lines.
   - \( a \) gradient 4, passing through \((0, 0)\)
   - \( b \) gradient \(-1\), passing through \((0, 1)\)
   - \( c \) gradient \(-\frac{3}{2}\), passing through \((0, -2)\)
   - \( d \) gradient \(\frac{2}{5}\), passing through \((0, -2)\)
   - \( e \) gradient \( \frac{5}{4} \), passing through \((-3, 4)\)
   - \( f \) gradient \(-\frac{3}{4}\), passing through \((4, -8)\)
   - \( g \) gradient \(-\frac{1}{3}\), passing through \((-1, -3)\)
   - \( h \) gradient \(-7\), passing through \((-6, 1)\)

3. **WE6** Find the equations for the line which passes through these points.
   - \( a \) \((1, 2), (2, 4)\)
   - \( b \) \((3, 5), (7, 11)\)
   - \( c \) \((0, 4), (3, 4)\)
   - \( d \) \((5, 0), (7, 2)\)
   - \( e \) \((2, 0), (5, 3)\)
   - \( f \) \((4, 20), (1, 5)\)
   - \( g \) \((-1, -3), (2, -11)\)
   - \( h \) \((3, 21), (5, 1)\)

4. Find the equation of each of the following straight lines.

   ![Graphs](image-url)
5 Find the equation of each of the following straight lines.

\[ a \]
\[ \begin{align*}
(4, 1) & \\
(1, 2) & \\
(5, 1) & \\
\end{align*} \]

\[ b \]
\[ \begin{align*}
(1, 7) & \\
(-1, -2) & \\
\end{align*} \]

\[ c \]
\[ \begin{align*}
\end{align*} \]

\[ d \]
\[ \begin{align*}
\end{align*} \]

6 Find the equation of each of a line:

\[ a \] parallel to the \( x \)-axis and with \( y \)-intercept 2
\[ b \] parallel to the line with equation \( y = x - 5 \) and with \( y \)-intercept \(-4\)
\[ c \] parallel to the line \( y = 2x + 1 \) and passing through the point \((1, 10)\)
\[ d \] passing through the origin and the point \((2, 7)\)
\[ e \] intersecting the \( x \)- and \( y \)-axes at \(-1\) and \(-4\) respectively.

7 \[ MC \] \[ a \] The equation of the line that passes through the points \((-1, 5)\) and \((1, 0)\) is:

\[ A \quad y = \frac{5}{2}x - \frac{5}{2} \]
\[ B \quad y = \frac{2}{5}x - \frac{2}{5} \]
\[ C \quad y = \frac{5}{2}x + \frac{5}{2} \]
\[ D \quad y = \frac{5}{2}x + \frac{5}{2} \]
\[ E \quad y = \frac{2}{5}x + \frac{2}{5} \]

\[ b \] The equation passing through the origin and parallel to the line \( y = 2x - 1 \) is:

\[ A \quad y = 0 \]
\[ B \quad y = -1 \]
\[ C \quad x = 0 \]
\[ D \quad y = -2x \]
\[ E \quad y = 2x \]

\[ c \] A line with a slope of \(-3\) passes through the point \((0, 3)\). Which of the following points does not lie on the same line?

\[ A \quad (-1, 6) \]
\[ B \quad (2, -3) \]
\[ C \quad (-4, 15) \]
\[ D \quad (3, -8) \]
\[ E \quad (5, -12) \]

\[ d \] A line passing through the point \((-1, 3)\) has a gradient of \(\frac{1}{2}\). Which of the following points lies on the same line?

\[ A \quad (-3, 1) \]
\[ B \quad (2, 4) \]
\[ C \quad (3, 5) \]
\[ D \quad (-2, 3) \]
\[ E \quad (4, 6) \]

8 Which one of the following statements about the line with equation \(12x - 4y = 0\) is not true?

\[ A \quad \text{The line passes through the origin.} \]
\[ B \quad \text{The line has a slope of 12.} \]
\[ C \quad \text{The line has the same slope as the line with the equation } 12x - 4y = 12. \]
\[ D \quad \text{The point } (1, 3) \text{ lies on the line.} \]
\[ E \quad \text{For this line, as } x \text{ increases } y \text{ increases.} \]

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### Sketching straight line graphs

At the beginning of this chapter we looked at the method of plotting straight line relations. Whether plotting manually or using technology, a table of points was generated first.

It is not necessary, however, to plot many points in order to obtain a graph of a straight line. In fact, only two points are needed, as only one straight line can be drawn through any two distinct points.

In this section we will consider two methods of sketching straight line graphs: the gradient–intercept method and the \(x-y\)-intercept method. The choice of the method usually depends on the form of the equation.
**Gradient–intercept method**

The method is convenient to use when the equation is in the gradient–intercept form (that is, \( y = mx + c \)). In this case one of the points (y-intercept) is known straight away and the other is obtained by using the gradient.

**Worked Example 9**

Use the gradient–intercept method to sketch the graph of \( y = \frac{2}{3}x + 4 \).

**THINK**

1. Write the equation of the line.
2. Identify the y-intercept (the value of \( c \) in the equation).
3. Write the formula of the gradient and equate it with the given gradient.
4. Identify rise and run.
5. Sketch the graph.

**WRITE**

- \( y = \frac{2}{3}x + 4 \)
- \( y \)-intercept: \( y = 4 \)
- \( m = \frac{\text{rise}}{\text{run}} \)
- \( m = \frac{2}{3} \)
- \( \text{rise} = 2; \text{run} = 3 \)

Note that if the gradient is a whole number, it can be turned into a fraction by placing it over 1.

For example, when \( m = 5 \), we can write \( m = \frac{\text{rise}}{\text{run}} = \frac{5}{1} \), hence, rise = 5 and run = 1.

Also, when the gradient is negative, the negative sign should be placed in the numerator of the fraction. For example, if \( m = -\frac{1}{4} \), we should write:

\[
m = \frac{\text{rise}}{\text{run}} = -\frac{1}{4}
\]

so rise = 1 and run = 4. In this case it is more like a ‘fall’ than a ‘rise’, as we have to go 1 unit down, not up.

**The x–y-intercept method**

The x–y-intercept method is convenient to use if the equation is written in the form

\[
ax + by = c \quad \text{or} \quad ax + by + c = 0.
\]

As the name suggests, the method involves finding the two points where a line intersects the x- and y-axes. Hence, if the question requires a graph showing x- and y-intercepts to be sketched, this method should be used regardless of the form of the equation.

At the point where the graph intersects the y-axis the x-value of the coordinates is 0 and at the point where the graph intersects the x-axis the y-value of the coordinates is 0.
WORKED EXAMPLE 10

Calculate the $x$- and $y$-intercepts and hence sketch the graph of $3x + 4y = 12$.

**THINK**

1. Alternatively, on the Main screen, complete the entry line as:
   \[
   \text{solve}(3x + 4y = 12, y) \mid x = 0
   \]
   Then press \(\text{Exe}\).

   Let $x = 0$.
   \[
   3(0) + 4y = 12
   \]
   \[
   4y = 12
   \]
   \[
   y = 3
   \]
   Coordinates of the $y$-intercept $(0, 3)$.

2. Alternatively, on the Main screen, complete the entry line as:
   \[
   \text{solve}(3x + 4y = 12, x) \mid y = 0
   \]
   Then press \(\text{Exe}\).

   Let $y = 0$.
   \[
   3x + 4(0) = 12
   \]
   \[
   3x = 12
   \]
   \[
   x = 4
   \]
   Coordinates of the $x$-intercept $(4, 0)$.

3. Plot the $x$- and $y$-intercepts on a set of axes and draw a straight line through the points.
Determining points of intersection of straight line graphs

When two straight lines are drawn on the same set of axes, they will intersect at one point (unless the lines coincide or are parallel).

The coordinates of the point of intersection of the two lines can be found either directly from the graph, or by using algebra, as shown in the following worked example.

WORKED EXAMPLE 11

Sketch the graphs of $2x + 3y = 12$ and $y = 4x - 10$ on the same set of axes. Determine the point of their intersection:

a from the graph  

b by using an algebraic method.

THINK

WRITE

a 1 Write the first equation.

2 Find the $y$-intercept by letting $x = 0$.

$2(0) + 3y = 12$

$3y = 12$

$y = 4$

Coordinates of the $y$-intercept: $(0, 4)$

Let $y = 0$

$2x + 3(0) = 12$

$2x = 12$

$x = 6$

Coordinates of the $x$-intercept: $(6, 0)$

3 Find the $x$-intercept by letting $y = 0$.

4 Write the second equation.

5 Find the $y$-intercept by letting $x = 0$.

$y = 4(0) - 10$

$y = -10$

Coordinates of the $y$-intercept: $(0, -10)$

Let $y = 0$

$0 = 4x - 10$

$4x = 10$

$x = 2.5$

Coordinates of the $x$-intercept: $(2.5, 0)$

6 Find the $x$-intercept by letting $y = 0$.

7 Sketch both lines on the same set of axes by plotting and joining their $x$- and $y$-intercepts.

8 Read the coordinates of the point of intersection directly from the graph.

b 1 Rearrange the first equation to make $y$ the subject.

$2x + 3y = 12$

$3y = 12 - 2x$

Point of intersection: $(3, 2)$
2 Write the second equation.

3 At the point of intersection, the values of \( x \) and \( y \) are the same for both equations. So equate the right-hand sides of the two equations and solve for \( x \).

4 Substitute the value of \( x \) into either of the two equations to find the value of \( y \).

5 Write the coordinates of the point of intersection of the two lines.

\[
\begin{align*}
\frac{y}{3} &= \frac{12}{3} - \frac{2}{3}x \\
y &= 4 - \frac{2}{3}x \\
y &= 4x - 10 \\
4 - \frac{2}{3}x &= 4x - 10 \\
4 + 10 &= 4x + \frac{2}{3}x \\
14 &= \frac{12}{3}x + \frac{2}{3}x \\
14 &= \frac{14}{3}x \\
x &= \frac{14}{3} + \frac{14}{3} \\
x &= \frac{14}{3} \times \frac{3}{14} \\
x &= 1
\end{align*}
\]

\[
\begin{align*}
y &= 4(3) - 10 \\
y &= 2 \\
\text{Point of intersection: } (3, 2)
\end{align*}
\]

Note: When we are finding the coordinates of the point of intersection of two lines graphically, the accuracy of our solution depends on the accuracy of the graphs. Therefore, it is always a good idea to check the answer obtained by substitution. This can be done by substituting the coordinates of the point of intersection into each equation and checking whether the result is a true statement. For instance, in the above worked example, the coordinates of the point of intersection of the lines \( 2x + 3y = 12 \) and \( y = 4x - 10 \) were (from the graph) found to be \( (3, 2) \).

To verify this, substitute these coordinates into each equation:

\[
\begin{align*}
2x + 3y &= 12 \\ 2(3) + 3(2) &= 12 \\
6 + 6 &= 12 \\
12 &= 12
\end{align*}
\]

\[
\begin{align*}
y &= 4x - 10 \\ 2 &= 4(3) - 10 \\
2 &= 12 - 10 \\
2 &= 2
\end{align*}
\]

Since both equations are true statements, the solution is correct.

The point of intersection of two straight line graphs can easily be found with the aid of a CAS calculator, as shown in the worked example below.
Write the solution.

On the Main screen, set up the two equations by tapping:
- 2D
- {}:

Complete the entry lines as shown in the screen and press \( \text{Enter} \).

Write the solution.

Write down the answer. You may choose to state the coordinates either as fractions or decimals.

From the graph, the point of intersection of \( y = 3x + 2 \) and \( y = 5 - x \) is \((0.75, 4.25)\).

Solving \( y = 3x + 2 \) and \( y = 5 - x \) simultaneously for \( x \) gives
\[
x = \frac{3}{4}; \quad y = \frac{17}{4}
\]

The coordinates of the point of intersection of \( y = 3x + 2 \) and \( y = 5 - x \) is \( \left( \frac{3}{4}, \frac{17}{4} \right) \) or \((0.75, 4.25)\).

**REMEMBER**

To sketch the graph of a straight line:
1. If the equation is in the form \( y = mx + c \), use the gradient–intercept method.
   When finding rise and run:
   (a) put \( m \) over 1 if it is a whole number
   (b) place the negative sign in the numerator of the fraction if \( m \) is negative.
2. If the equation is in the form \( ax + by = c \) (or \( ax + by + c = 0 \)), or if you are asked to show intercepts on the axes, use the \( x \)-\( y \)-intercept method.
   (a) The \( x \)-intercept occurs when \( y = 0 \).
   (b) The \( y \)-intercept occurs when \( x = 0 \).
3. If everything else fails, you can always resort to plotting! Just find the coordinates of any two points, plot them, and join with a straight line.
4. When two straight lines are drawn on the same set of axes, they will intersect at one point (unless the lines coincide or are parallel). The coordinates \((x, y)\) of the point of intersection of the two lines can be read directly from the graph.
5. The coordinates of the point of intersection can also be found using algebra as follows.
   (a) Write each of the two equations in \( y = mx + c \) form.
   (b) Equate the right-hand sides of the two equations and solve for \( x \).
   (c) Substitute the value of \( x \) into either of the two equations and solve for \( y \).
EXERCISE 3E

Sketching straight line graphs

1. Which of the following equations are in the gradient-intercept form?
   a) \( y = 3x - 2 \)
   b) \( y + x = 12 \)
   c) \( 2y + 3x - 4 = 0 \)
   d) \( y = -6x \)
   e) \( x - 5 = y \)
   f) \( y = 2 - 5x \)
   g) \( 2x - y = 3 \)
   h) \( -x + 2y + 1 = 0 \)
   i) \( y = \frac{2}{3}x + 1 \)

2. Sketch each of the following graphs using the gradient-intercept method.
   a) \( y = 5x + 2 \)
   b) \( y = \frac{3}{4}x + 1 \)
   c) \( y = 2x \)
   d) \( y = x - 5 \)
   e) \( y = -3x \)
   f) \( y = -2x + 4 \)
   g) \( y = \frac{2}{3}x + 3 \)
   h) \( y = -0.5x - 2 \)
   i) \( y = 1 - \frac{1}{4}x \)

3. When graphing \( y = 2 - \frac{4}{5}x \) using the gradient-intercept method, from the y-intercept we should move:
   A) 5 units up and 3 units to the left
   B) 5 units down and 3 units to the right
   C) 3 units up and 5 units to the right
   D) 3 units down and 5 units to the left
   E) 3 units down and 5 units to the right

4. The graph of \( y = \frac{4}{5}x - 2 \) passes through the point:
   A) (3, 5)
   B) (3, 6)
   C) (4, 5)
   D) (3, 2)
   E) (4, 1)

5. Give the coordinates of:
   i) the y-intercept
   ii) the x-intercept
   for each of the following graphs.

6. Calculate the x- and y-intercepts of each linear equation and hence sketch their graphs.
   a) \( x + y = 3 \)
   b) \( y - x = 5 \)
   c) \( 2x + y = 1 \)
   d) \( 3x - 4y = -12 \)
   e) \( x + 5y - 2 = 0 \)
   f) \( 3y - 2x + 8 = 0 \)
   g) \( y = 2x - 5 \)
   h) \( y = \frac{2}{3}x + 7 \)
   i) \( y = -1.5x + 3 \)
   j) \( y = -4x + 1.5 \)

   Check your answers with a CAS calculator.

7. Sketch the graphs of each of the following pairs of equations on the same set of axes. In each case, determine the point of intersection:
   i) from the graph
   ii) by using an algebraic method.
   a) \( y = 2x + 4 \) and \( y = x + 5 \)
   b) \( y = -3x + 1 \) and \( y = 2x + 6 \)
   c) \( y = x + 4 \) and \( y = 6 - x \)
   d) \( 2x - y = 2 \) and \( y = x \)
8. For the following pairs of equations use a CAS calculator: to determine the coordinates of the point of intersection:
   i) from the graph and
   a) \(3x + 4y = 2 \) and \( y = 3x + 7 \)
   b) \( 4y + 2x = 4 \) and \( 3y - x = 8 \)
   ii) algebraically.
   b) \( 2y - 3x = 4 \) and \( x + y = 7 \)
   d) \( 5x - 2y = 1 \) and \( 2x + 3y = 11 \)

9. Which of the following pairs of straight lines intersect at the point \((1, -2)\)?
   A) \( x + y = 1 \) and \( 2x + 3y = -1 \)
   B) \( 3x - 4y = -5 \) and \( 2y + x = -3 \)
   C) \( 3y + x = 5 \) and \( 4x + 2y = 8 \)
   D) \( 2x - y = 4 \) and \( 2y - x = -5 \)

10. Which of the following represents the coordinates of the point of intersection of the two lines shown at right?
    A) \((-2, -1)\)  B) \((-2, -2)\)
    C) \((-1, -2)\)  D) \((-1, -3)\)
    E) \((-3, -1)\)

---

**3F Application of linear modelling**

Many real-life situations can be modelled by a linear function and/or graph. Once the equation or rule has been established, it can be used to make predictions or calculate specific values as required.

**WORKED EXAMPLE 13**

The Avanti car rental company charges $80 for the hire of a car plus 22 cents per kilometre travelled.

a) How much will it cost to travel 300 kilometres?

b) Determine the cost ($C$) equation for a distance of \(x\) kilometres.

c) Graph the function for \(0 \leq x \leq 1000\).

d) If the final cost was $245, what distance was covered during the hiring period?

**THINK**

a) 1) Express money in the same units.
   2) Find the cost of travelling 300 kilometres.
   3) Add the charge for the hire of a car to find the total cost.

b) 1) Write the cost of travelling any distance \(x\).
   2) Add the charge for the hire to find the total cost.

**WRITE**

a) 22 cents = $0.22
   Cost of travelling 300 km at $0.22 per km:
   \[0.22 \times 300 = 66\]
   Total cost:
   \[C = 80 + 66 = $146\]

b) Cost of travelling \(x\) km at $0.22 per km:
   \[0.22 \times x\]
   Total cost:
   \[C = 0.22x + 80\]

c) For \(0 \leq x \leq 1000\):
   when \(x = 0\), \[C = 0.22 \times 0 + 80 = 80\]
   when \(x = 1000\), \[C = 0.22 \times 1000 + 80 = 300\]
   So the points are \((0, 80)\) and \((1000, 300)\).
Draw the set of axes with distance on the horizontal axis and cost on the vertical axis, plot the two points and join them with the straight line. Note that since neither distance nor cost can be negative, we only need the first quadrant.

d Substitute $245 for $C$ in the general equation of the cost and solve for $x$.

d $C = 0.22x + 80$

When $C = 245$:

\[
245 = 0.22x + 80
\]

\[
165 = 0.22x
\]

\[
x = \frac{165}{0.22}
\]

\[
= 750
\]

Hence, 750 kilometres were covered.

In the previous example both the gradient and the $y$-intercept were known, so it was a simple matter of substituting given values into the general rule $y = mx + c$ to establish the equation of the cost. Sometimes, however, we know the gradient only, or the $y$-intercept only, and sometimes neither of them is given. In such cases there will always be some extra information describing the relation between the variables which will enable you to find the equation.

**WORKED EXAMPLE 14**

A carpet cleaning company charges $35 to steam-clean one bedroom and $55 for three bedrooms. Find a linear model to express the charge ($C$) for steam-cleaning a place containing $n$ bedrooms.

**THINK**

1. Identify the dependent and independent variables and write them as an ordered pair.

2. Write the information given in the problem as the coordinates of two points.

3. To find the equation of the line passing through two given points first write the general linear equation.

4. Find the gradient.

5. Substitute the gradient and coordinates of any of the two points (say, the first one) into the general equation and solve for $c$.

6. Substitute the value of $c$ into $y = 10x + c$.

7. Rewrite the equation, using the pronumerals in question.

**WRITE**

Number of bedrooms $n$ — independent variable
Cost of cleaning $C$ — dependent variable
Hence, $(n, C)$

(1, 35) and (3, 55)

\[y = mx + c\]

\[m = \frac{55 - 35}{3 - 1}\]

\[= \frac{20}{2}\]

\[= 10\]

Using $m = 10$ and (1, 35):

\[35 = 10 \times 1 + c\]

\[c = 35 - 10\]

\[= 25\]

\[y = 10x + 25\]

The equation relating the charge for cleaning and the number of bedrooms is $C = 10n + 25$. 

140 Maths Quest 11 Standard General Mathematics for the Casio ClassPad
Many real-life situations involve comparing two (or more) sets of variables whose relationships are linear. For example, when renting a car while on holidays, we might need to choose between two different deals: one charging a flat rate of $60 a day, the other charging 90 cents for every kilometre travelled. In this case, the variables are the cost of renting and the distance travelled. To compare the two deals (and hence, to help make a decision), linear graphs can be used.

WORKED EXAMPLE 15

Opus telephone company offers two packages for their customers.
Package 1: A $21 monthly service fee plus 14 cents per call.
Package 2: A $7 monthly service fee plus 28 cents per call.

a For each package, find the equation to represent the cost, $C$, of making $n$ calls per month.
b Graph both equations on the same set of axes for $0 \leq n \leq 200$.
c How many calls would you need to make per month for the cost of both packages to be the same?
d For what number of calls would Package 1 be cheaper?

THINK

a 1 Write the equation to represent the monthly cost for Package 1. It consists of the cost of making $n$ calls at $0.14$ per call ($0.14 \times n$) and the fixed service fee of $21$.

2 Write the equation to represent the monthly cost for Package 2. It consists of the cost of making $n$ calls at $0.28$ per call ($0.28 \times n$) and the fixed service fee of $7$.

WRITE

a $C_1 = 0.14n + 21$

$C_2 = 0.28n + 7$

b $0 \leq n \leq 200$:
when $n = 0$, $C_1 = 0.14 \times 0 + 21 = 21$
when $n = 200$, $C_1 = 0.14 \times 200 + 21 = 49$
So the points are (0, 21) and (200, 49).

When $n = 0$, $C_2 = 0.28 \times 0 + 7 = 7$
when $n = 200$, $C_2 = 0.28 \times 200 + 7 = 63$
So the points are (0, 7) and (200, 63).

3 Draw a set of axes with the number of calls on the horizontal axis and the cost on the vertical axis. For each equation, plot the end points and join them with straight lines. Label your lines with the equations.
The point of intersection of the two lines is the point where cost of making \( n \) calls is the same for both packages. So read the coordinates of the point of intersection from the graph.

Verify the solution algebraically. On the Main screen, set up the two equations by tapping:
- \( \text{2D} \)
- \( \text{1} \)

Complete the entry lines as shown in the screen and press \( \text{EX} \).

Write the solution.

Both the algebraic and graphical solutions are the same, so state your conclusion.

We require \( C_1 < C_2 \). Graphically this means that the line \( C_1 \) must be below the line \( C_2 \). This occurs to the right of the point of intersection (that is, when \( n > 100 \)).

From the graph, the point of intersection is \((100, 35)\).

Solving \( c = 0.14n + 21 \) and \( c = 0.28n + 7 \) for \( n \) and \( c \) gives \( n = 100 \) and \( c = 35 \).

The costs of the two packages are the same ($35) when 100 calls are made.

Package 1 will be cheaper for any number of calls above 100.

**Line segment graphs**

Sometimes the relationship between two variables is linear, but follows different rules for different intervals. The graph of such a relationship would then not be a single straight line, but two or more segments of straight lines, whose equations differ. Such a graph is referred to as a *line segment graph*.

Line segment graphs can be easily constructed by plotting and joining the starting point and the finishing point of each segment.

A typical situation that can be represented on a line segment graph is a journey, where different sections of the trip are travelled at different speeds. This is illustrated in the following worked example.

**WORKED EXAMPLE 16**

Maya and Joseph decided to go to their holiday house for the weekend. Maya drove for the first three-quarters of an hour at an average speed of 60 km/h. After stopping for 15 minutes to rest, they swapped seats and Joseph drove the rest of the way. He drove for 1 hour and averaged 90 km/h.

a Draw a graph to represent the journey. Place time \( (t) \) on the horizontal axis and distance travelled \( (D) \) on the vertical axis.

b Find the total distance that Maya and Joseph drove.
c Find the total time it took for Maya and Joseph to reach their destination.
d What was the average speed of the entire trip?

**THINK**

1. The first section of the trip began at (0, 0), since both time and distance travelled are 0. Find the end point of the first section after travelling for $\frac{3}{4}$ of an hour with an average speed of 60 km/h.

2. The second section of the trip began where the first section ended; that is, at (0.75, 45) and ended 15 minutes later. Since there was no movement during this period, the value of $D$ at the finishing point will remain unchanged.

3. The third section of the trip began where the second section ended; that is, at (1, 45). Find the end point of the third section after travelling for 1 hour with the average speed of 90 km/h.

4. Draw a set of axes and label them. Plot the starting and finishing points of each section of the trip and join them with straight line segments.

**WRITE/DRAW**

a First section:
Starting point: (0, 0)
Time travelled = $\frac{3}{4}$ of an hour
Speed = 60 km/h
Distance travelled = $\frac{3}{4} \times 60 = 45$ km
So the finishing point is (0.75, 45).

Second section:
Starting point: (0.75, 45).
Time resting = 15 minutes ($\frac{1}{4}$ of an hour)
Distance travelled = 0
At the finishing point, $t = 1.25$ and $D = 45 + 0 = 45$.
So the finishing point is (1, 45).

Third section:
Starting point: (1, 45)
Time travelled = 1 hour
Speed = 90 km/h
Distance travelled = $1 \times 90 = 90$ km
At the finishing point, $t = 2$ and $D = 45 + 90 = 135$.
So the finishing point is (2, 135).

b At the beginning of the journey $D = 0$ and at the end point of the last section of the trip, $D = 135$, so the total distance travelled was 135 km.

c At the beginning of the journey $t = 0$ and at the end point of the last section of the trip, $t = 2$, so the total time spent travelling to the holiday house was 2 hours.

b Total distance travelled = 135 km
c Total time = 2 hours
To find the average speed of the trip, divide the total distance travelled by the total time taken to get to the destination.

\[
\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}
\]

\[
= \frac{135}{2}
\]

\[
= 67.5 \text{ km/h}
\]

**REMEMBER**

1. When modelling real-life situations with linear functions:
   (a) identify the independent and dependent variables first
   (b) when graphing, place the independent variable on the horizontal axis and dependent variable on the vertical axis
   (c) take care to use the variables given in the problem rather than \( x \) and \( y \).
2. When finding the coordinates of the point of intersection of two lines, always verify algebraically the solutions obtained from the graph.
3. A line segment graph contains two or more segments of straight lines, whose equations differ.
4. To construct a line segment graph, plot and join the starting and finishing point of each segment.

**EXERCISE 3F**

**Application of linear modelling**

1. **WE13** Jake and Voula Catering Enterprises charge $250 for the hire of the function centre plus $8 per head for the spit roast.
   a) How much will it cost for a party of 50 people?
   b) Determine the cost, \( C \), equation for a party of \( n \) people.
   c) Graph the function for \( 0 \leq n \leq 250 \).
   d) If the cost for a function was $1050, how many people were expected to attend?

2. A local football club decided to hire a bus to the Grand Final. The bus company charges $200 for a 45-seater bus and $5 per person.
   a) How much will it cost if every seat is occupied?
   b) Construct an equation for the total cost, \( C \), if \( n \) people catch the bus.
   c) Graph the equation.
   d) What does the \( y \)-intercept represent?
   e) What does the gradient represent?
   f) If the final cost was $380, how many people went by bus?

3. **MC** A builder’s fee, \( C \) dollars, can be determined from the rule \( C = 60 + 55n \), where \( n \) represents the number of hours worked.
   According to this rule, the builder’s fee will be:
   A $60 for 1 hour of work    B $110 for 2 hours of work    C $500 for 8 hours of work
   D $550 for 10 hours of work  E $1150 for 10 hours of work

4. The temperature \( T \) in the room \( t \) minutes after the air-conditioner was turned on is given by the graph at right.
   a) What was the initial temperature in the room?
   b) By how many degrees does the temperature drop each minute?
   c) Write the rule connecting temperature, \( T \), and time, \( t \), and specify the domain.
d What was the temperature in the room 2 minutes after the air-conditioner was turned on?
e In how many minutes did the temperature drop to 21 degrees?

5 WE14 An electrician charges $70 for a job which takes 2 hours to complete and $92.50 for
3.5 hours of work.

a Find a linear model to express the charge, \(C\), for \(t\) hours of work.
b Find the maximum daily earnings of the electrician if she is not prepared to work more
than 8 hours on any given day.
c Find the time taken to complete the job if the charge was $73.75.

6 A long-distance truck driver is travelling towards home. After 1.5 hours of driving, he is
260 km from home and 2 hours later he is 140 km from his destination.

a Find a linear model to express the distance from home, \(D\), at any time, \(t\), from the
beginning of the journey.
b Sketch the graph of the relation.
c How far away from home was the driver when he started the journey?
d What is the speed of the truck?
e How long did it take to complete the journey? (Give the answer in hours and
minutes.)

7 WE15 Ausat Telecommunications charges $60 per quarter plus 35 cents for every call. Their
rivals, Tetcom Telecommunications charge $45 per quarter plus 45 cents for every call.

a For Ausat, find the equation relating the total quarterly telephone bill, \(T\), to the
number of telephone calls, \(n\).
b For Tetcom, find the equation relating the total quarterly telephone bill, \(T\), to the
number of telephone calls, \(n\).
c Graph both equations on the same set of axes.
d Use your graph to find when the quarterly cost is the same for both companies.
e If you made, on average, only 120 calls per quarter, which company would be the better choice and by how much?

8 The Rave T-Shirt Company produces T-shirts at a cost of $6 each after an initial setting up cost
of $300.

a How much does it cost Rave to produce 100 T-shirts?
b Find an equation that describes the cost, \(C\), to produce \(n\) T-shirts.
c Graph the equation.
d The selling price, \(S\), for each T-shirt is $10.80. Find the equation expressing \(S\) in terms of
\(n\) T-shirts.
e Sketch the graph on the same set of axes you used in part c above.
f From the graph, find the number of T-shirts needed to break even.
g If an order were completed for 130 T-shirts, how much profit would Rave make?

9 MC The cost of manufacturing a number of frying pans consists of a fixed cost of $400
plus a cost of $50 per frying pan.
The manufacturer could break even by selling:

A 10 frying pans at $90 each
C 15 frying pans at $60 each
E 20 frying pans at $50 each
B 10 frying pans at $45 each
D 15 frying pans at $30 each

[©VCAA 2006]
10 Metros Hire Car Company offers two daily rate packages.  
Package 1: An up-front charge of $55 plus 26 cents per kilometre.  
Package 2: An up-front charge of $100 per day, unlimited kilometres.  
   a Find an equation for the cost, $C$, of each package per day.  
   b Graph each equation on the same set of axes.  
   c How many kilometres would you need to travel per day for the cost of both packages to be the same?  
   d For what distances would Package 1 be cheaper?

11 Two friends, Michael and Julia, work in a large department store. Michael sells men’s apparel and earns $520 per week. Julia is in the ‘white goods’ department, selling refrigerators, dishwashers, washing machines and so on. She earns a $200 per week retainer, plus 8% of the total amount of her weekly sales.  
   a Find the equation to represent the amount, $$A$$, earned by each friend per week. (Use $$s$$ to denote the total value of sales.)  
   b Graph each equation on the same set of axes.  
   c What should be the total value of goods sold by Julia so that her weekly earnings will be the same as Michael’s?  
   d During one week, Julia sells white goods to the total value of $12,500. Whose earning would be greater for that week, and by how much?

12 A local karate club, ‘Quick Kick’ charges a $120 joining fee and $6 for every lesson attended. Their competitors across the road, the ‘Super Kids’ club, charge $10 per lesson, but there is no joining fee.  
   a Find the equation to represent the total cost $$C$$ of attending $$n$$ lessons for each club.  
   b Graph each equation on the same set of axes.  
   c How many lessons would need to be attended, for the total tuition costs to be the same for both clubs?  
   d If Alex intends to go to karate lessons once a week for an entire year, which club should he choose (provided that his decision is based on the total cost of tuition only)?

13 Last Easter, Nathan and Rachel travelled to Lorne. Nathan drove for the first one and a half hours at an average speed of 60 km/h. They then stopped at a service station for 30 minutes to fill the petrol tank and rest. Rachel then drove for 1 hour and 15 minutes, and averaged 80 km/h.  
   a Draw a graph to represent the journey. Place time on the horizontal axis and distance travelled on the vertical axis.  
   b Find the total distance that Nathan and Rachel drove.  
   c Find the total time it took for Nathan and Rachel to travel to Lorne.  
   d What was the average speed of the entire trip?
Questions 14 to 16 refer to the information below.
The graph at right shows a day trip of a cyclist.

14 MC Which section or sections of the graph indicate that the cyclist was moving at a speed of 10 km/h?
A AB  B BC  C CD  D DE  E AB and DE

15 MC What was the speed of the cyclist for the interval CD?
A 5 km/h  B $\frac{6}{3}$ km/h  C 10 km/h  D 15 km/h  E 20 km/h

16 MC What was the average speed of the entire trip?
A 10 km/h  B 11 km/h  C $11\frac{2}{3}$ km/h  D $12\frac{1}{2}$ km/h  E $12\frac{2}{3}$ km/h

17 The Cybercheap company offers discounts on all of its products purchased over the Internet. The conditions are as follows:
1. a 10% discount is given on all purchases to the total value up to $500 and
2. a 7% discount is given on any amount in excess of $500.
   a Draw a line segment graph to show the discount, $D$, (in dollars) for purchase, $P$, of any size up to $1000. (Place discount on the vertical axis and purchase price on the horizontal axis.)
   b Find the equation for each segment of the graph.
   c Find the discount for a purchase to the total value of:
      i $375  
      ii $820.

18 Every Saturday morning, Gary goes delivering newspapers in his neighbourhood. He is paid $0.03 for each newspaper delivered up to (and including) 300, and $0.05 for every newspaper delivered in excess of 300.
   a Draw a line segment graph to show Gary’s total earnings, $E$, (in dollars) for delivering any number of newspapers, $n$, up to 500. (Place earnings on the vertical axis and number of newspapers delivered on the horizontal axis.)
   b Find the equation for each segment of the graph.
   c Calculate the total amount that Gary will earn for delivering:
      i 221 newspapers  
      ii 475 newspapers.

3G Line of best fit (by eye)

When data involving two variable quantities are graphed against each other the resulting scatterplot may indicate an approximate linear relationship.

To establish a linear model, a line of best fit (by eye) is drawn through the scatterplot so that approximately half the data points lie either side of the line and are situated close to the line.

WORKED EXAMPLE 17

The table below gives the practice exam marks (in %) and actual exam marks obtained by 8 students.

<table>
<thead>
<tr>
<th>Practice exam mark (%)</th>
<th>48</th>
<th>62</th>
<th>70</th>
<th>59</th>
<th>82</th>
<th>68</th>
<th>80</th>
<th>56</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual exam mark (%)</td>
<td>55</td>
<td>78</td>
<td>88</td>
<td>54</td>
<td>90</td>
<td>62</td>
<td>89</td>
<td>68</td>
</tr>
</tbody>
</table>

a Draw a scatterplot to represent the data and draw a line of best fit (by eye).

b Determine the equation of the line of best fit.

c Use your equation to estimate the actual exam mark of the student whose practice exam mark was 75%. (Give your answer to the nearest percentage.)

d If a student obtained 88% on the actual exam, use the equation to estimate her practice exam mark. (Give your answer to the nearest percentage.)
**THINK**

a Plot the points from the table and draw a line of best fit (by eye).

b 1 Choose two points on the line that are not too close together. (In this case the line passes through two data points from the table, so it is convenient to choose these points.)

2 Calculate the gradient.

3 Substitute the value of the gradient and the coordinates of any of the two points (say, the first one) into the general equation to find c.

4 State the equation of the line of best fit in terms of the variables in question.

c Substitute 75 for practice exam mark into equation of the line of best fit to find the actual exam mark. Round off your answer to the nearest percentage.

d Substitute 88 for actual exam mark into equation of the line of best fit to find the practice exam mark. Round off your answer to the nearest percentage.

**WRITE**

a

 Pradesh exam mark (%)

Actual exam mark (%)

<table>
<thead>
<tr>
<th>Practice exam mark (%)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual exam mark (%)</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>100</td>
</tr>
</tbody>
</table>

b Let (48, 55) be \((x_1, y_1)\) and (80, 89) be \((x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{89 - 55}{80 - 48} = \frac{34}{32} = 1.0625
\]

Substitute \(m = 1.0625\) and \((48, 55)\) into \(y = mx + c\):

\[
55 = 1.0625 \times 48 + c \quad 55 = 51 + c \quad c = 55 - 51 \quad c = 4
\]

Hence, \(y = 1.0625x + 4\)

Therefore, the equation of the line of best fit is:

Actual exam mark = 1.0625 \times \text{practice exam mark} + 4

c Practice exam mark = 75

Actual exam mark = 1.0625 \times 75 + 4

= 83.6875

= 84%

d Practice exam mark = 88

84 = 1.0625 \times \text{practice exam mark}

Practice exam mark = \frac{84}{1.0625}

= 79.0588

= 79%
A CAS calculator can be used to assist in constructing of the scatterplot and the line of best fit as shown in the example below.

**WORKED EXAMPLE 18**

The following results were obtained during an experiment.

<table>
<thead>
<tr>
<th>$r$ (cm)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$ (g)</td>
<td>25.0</td>
<td>31.5</td>
<td>36.3</td>
<td>41.9</td>
<td>47.6</td>
<td>52.8</td>
<td>58.6</td>
<td>63.7</td>
<td>69.6</td>
<td>75</td>
</tr>
</tbody>
</table>

a Construct a scatterplot and draw a line of best fit (by eye) using a CAS calculator.

b Determine the equation of the line of best fit.

c Estimate the value of $w$ (g) if $r = 6.4$ centimetres. Give answer correct to 1 decimal place.

d If a mass of 80 g was recorded, estimate the length $r$ (cm). Give answer correct to 1 decimal place.

**THINK**

a 1 On the Statistics screen, label list1 as ‘r’ and enter the values from the table.
Label list2 as ‘w’ and enter in the corresponding values.

2 To plot the points, tap:
- **SetGraph**
- **Setting**
  Set:
  - Type: Scatter
  - XList: main\r
  - YList: main\w
  - Freq: 1
  - Mark: Square
- **Set**
- **Calc**
  **Linear Reg**
  Set:
  - XList: main\r
  - YList: main\w
  - Freq: 1
- **OK**

**WRITE/DISPLAY**

a 1 Copy the equation from the screen into your workbook.

b Equation of the line of best fit is:
$y = 5.53x + 20$
Replace $x$ and $y$ with variables in question.

Substitute $r = 6.4$ into the equation of the line to find $w$.

Substitute $w = 80$ into the equation of the line to find $r$.

When $r = 6.4$
\[ w = (5.53)(6.4) + 20 \]
\[ = 55.392 \]
\[ = 55.4 \text{ g} \]

When $w = 80$
\[ 80 = 5.53r + 20 \]
\[ 60 = 5.53r \]
\[ r = \frac{60}{5.53} \]
\[ = 10.8499 \]
\[ \approx 10.8 \]

The estimated length is 10.8 cm.

**REMEMBER**

1. When drawing a line of best fit by eye through a scatterplot, try to ensure that about half of all points lie on each side of the line and are close to it.

2. When finding the equation of the line of best fit, select two points that are not too close to each other.

**EXERCISE**

**3G** Line of best fit (by eye)

1. The results (in percentages) obtained by 8 students studying both English and Italian are shown in the table below.

<table>
<thead>
<tr>
<th>English</th>
<th>26</th>
<th>39</th>
<th>47</th>
<th>52</th>
<th>61</th>
<th>74</th>
<th>82</th>
<th>96</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italian</td>
<td>17</td>
<td>45</td>
<td>57</td>
<td>53</td>
<td>56</td>
<td>74</td>
<td>87</td>
<td>92</td>
</tr>
</tbody>
</table>

a) Plot a scatter diagram and draw a line of best fit (by eye).
b) Determine the equation of the line of best fit.
c) Estimate the result for Italian, if a student gains 90% for English.
d) If a student achieves 60% for Italian, estimate the student’s score for English.

2. The ages (in years) of small 4-cylinder cars and their values are given in the following table.

<table>
<thead>
<tr>
<th>$t$ (time — years)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$ (value — $)</td>
<td>20000</td>
<td>16750</td>
<td>13250</td>
<td>11600</td>
<td>10500</td>
<td>9200</td>
</tr>
</tbody>
</table>

a) Make a scatterplot of the data and draw a line of best fit (by eye).
b) Find the equation of the line of best fit.
c) Estimate the value of a 6-year-old, 4-cylinder car.
d) In 10 years’ time, what would be the expected value of a 4-cylinder car? Comment on your answer.
3 The height (cm) and weight (kg) of each student in a Physical Education class was recorded. Some of the results are shown in the following table.

<table>
<thead>
<tr>
<th>h (height — cm)</th>
<th>160</th>
<th>163</th>
<th>168</th>
<th>172</th>
<th>175</th>
<th>184</th>
<th>184</th>
<th>186</th>
</tr>
</thead>
<tbody>
<tr>
<td>w (weight — kg)</td>
<td>64</td>
<td>56</td>
<td>69</td>
<td>67</td>
<td>70</td>
<td>73</td>
<td>67</td>
<td>80</td>
</tr>
</tbody>
</table>

a Plot weight against height and draw a line of best fit (by eye).
b Find the equation of the line of best fit.
c Estimate the weight of a student whose height is 170 cm.
d If the student’s weight was 65 kg, estimate the student’s height.

4 A physics student recorded the speed of a body (v cm/s) at one second intervals. The results are shown below.

<table>
<thead>
<tr>
<th>t (time)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>v (speed)</td>
<td>5.1</td>
<td>10.6</td>
<td>15.7</td>
<td>20.1</td>
<td>26.2</td>
<td>31.5</td>
</tr>
</tbody>
</table>

a Construct a scatterplot and draw a line of best fit (by eye) using a CAS calculator.
b Find the equation of the line of best fit.
c Estimate the speed when t = 4.3 seconds.
d Predict how long it will take to reach a speed of 40 cm/s.

5 The following table gives the number of employees and the number of investor accounts for eight investment companies.

<table>
<thead>
<tr>
<th>e (number of employees)</th>
<th>34</th>
<th>46</th>
<th>46</th>
<th>50</th>
<th>68</th>
<th>146</th>
<th>272</th>
<th>517</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (investor accounts, '000s)</td>
<td>41.8</td>
<td>27</td>
<td>47.4</td>
<td>41.3</td>
<td>79.2</td>
<td>98.6</td>
<td>163.1</td>
<td>303</td>
</tr>
</tbody>
</table>

a Create a scatterplot and draw a line of best fit (by eye) using a CAS calculator.
b Find the equation of the line of best fit.
c Estimate the number of investor accounts when a company has 100 employees.
d Estimate the number of employees in a company with 350 000 investor accounts.
Plotting straight line graphs

- To plot a straight line graph, construct a table of values, plot the points and join them with a straight line.
- To sketch a straight line graph when the equation is in the form \( y = mx + c \), use the gradient–intercept method:
  \[ m = \frac{\text{rise}}{\text{run}}; \quad c \text{ is the } y\text{-intercept}. \]
- To sketch a straight line graph when the equation is in the form \( ax + by = c \) or \( ax + by + c = 0 \), or when the intercepts with the axes must be shown, use the \( x\)–\( y\)-intercept method:
  1. find the \( y\)-intercept by letting \( x = 0 \)
  2. find the \( x\)-intercept by letting \( y = 0 \)
  3. plot the intercepts and join them with a straight line.

Finding the gradient of a straight line

- The gradient of the line joining points \( A(x_1, y_1) \) and \( B(x_2, y_2) \) is given by
  \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

Determining the equation of a straight line

- Given the gradient and \( y\)-intercept:
  substitute the values of the gradient and \( y\)-intercept into the equation instead of \( m \) and \( c \) respectively.
- Given the gradient and one point:
  substitute the gradient and the coordinates of the point into the equation to find the value of \( c \).
- Given two points:
  find the gradient first using the coordinates of the two points; find the value of \( c \) using the gradient and either of the two points.
- Given that the line is parallel to another line:
  parallel lines have the same gradient; therefore, identify the value of \( m \) by taking the gradient of the line, the equation for which is given.

Relating equations and graphs to values of \( m \) and \( c \)

- In the equation \( y = mx + c \), \( m \) is the gradient and \( c \) is the \( y\)-intercept.
  1. The gradient (\( m \))
     (a) If \( m < 0 \), the line slopes down to the right.
     (b) If \( m = 0 \), the line is horizontal (parallel to the \( x\)-axis).
     (c) If \( m > 0 \), the line slopes up to the right.
     (d) The value of \( m \) indicates the steepness of the line: the larger \( m \) is, the steeper the line.
     (e) Lines with the same value of \( m \) are parallel.
  2. \( c \) translates the line \( c \) units along the \( y\)-axis.
     (a) If \( c < 0 \), the line is moved down along the \( y\)-axis.
     (b) If \( c = 0 \), the line passes through the origin.
     (c) If \( c > 0 \), the line is moved up along the \( y\)-axis.

Finding the point of intersection of two lines

- When two straight lines are drawn on the same set of axes, they will intersect at one point (unless the lines coincide or are parallel). The coordinates \( (x, y) \) of the point of intersection of the two lines can be read directly from the graph.
The coordinates of the point of intersection can also be found using algebra as follows.
1. Write each of the two equations in $y = mx + c$ form.
2. Equate the right-hand sides of the two equations and solve for $x$.
3. Substitute the value of $x$ into either of the two equations and solve for $y$.

**Line segment graphs**

- A line segment graph contains two or more segments of straight lines, whose equations differ.
- To construct a line segment graph, plot and join the starting and finishing points of each segment.

**Line of best fit (by eye)**

- When the scatterplot indicates a linear relation, to establish a linear model between the two variables we draw a line through a scatterplot and find its equation.
- The line is drawn by eye so that about half of all points lie on each side of the line and are close to it.
CHAPTER REVIEW

MULTIPLE CHOICE

1 To plot $6x - 3y = 12$, the following table of values is constructed:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-2</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

The missing values are:
A $x = 4, y = 8$
b $x = 4, y = 10$
C $x = 5, y = 8$
d $x = 5, y = 10$
E $x = 5, y = 12$

2 The gradient of the line passing through the points $(2, 4)$ and $(3, y)$ is $-2$. The value of $y$ is:
A 6  B 4  C 2  D 0  E -2

3 The gradient of the line drawn is:
A $\frac{4}{3}$  B $\frac{3}{4}$  C $\frac{4}{3}$  D 3  E 4

4 The equation of the line which passes through the points $(3, -3)$ and $(1, 1)$ is:
A $y = 2x - 3$
b $y = -2x + 3$
c $y = 2x + 3$
d $y = -2x - 3$
E $y = x + 2$

5 The line parallel to $y = 4x - 5$ is:
A $y = 2x - 5$
b $4x + y + 2 = 0$
c $2y = 4x - 10$
d $4x - y - 1 = 0$
E $y = x + 5$

6 The graph of $2x - y - 2 = 0$ is:
A
B
C
D
E

7 The equation $3x + 5y - 15 = 0$ transposed into the form $y = mx + c$ is:
A $y = \frac{-3}{5}x + 3$
b $y = \frac{3}{5}x + 15$
c $y = \frac{3}{5}x + 3$
d $y = \frac{3}{5}x - 3$
E $y = 15 - \frac{3}{5}x$

8 Which of the following is not true of $y = -3x + \frac{1}{2}$?
A The line has a steeper slope than $y = x$.
b The $y$-intercept is at $y = -\frac{1}{2}$.
c $y = x + \frac{1}{2}$ has the same $y$-intercept.
d The line $y = -3x - 1$ is parallel.
e The point $(1, 2.5)$ lies on the line.

9 To sketch the graph of $y = 5 - \frac{2}{3}x$ using the gradient–intercept method, beginning from the $y$-intercept, we need to move:
A 2 units up and 3 units to the right
B 2 units down and 3 units to the right
C 2 units down and 3 units to the left
D 3 units down and 2 units to the right
E 3 units down and 2 units to the left

10 The line with equation $3x - 4y = 12$ intersects the axes at the points:
A $(3, 0), (0, -3)$  B $(3, 0), (0, 4)$  C $(4, 0), (0, -3)$  D $(-3, 0), (0, -4)$  E $(-4, 0), (0, 3)$

11 The coordinates of the point of intersection of the lines with equations $4x + 2y = 6$ and $y = 5x - 11$ are:
A $(1, 1)$  B $(2, 1)$  C $(-2, 1)$  D $(2, -1)$  E $(-2, -1)$

12 The point of intersection of two lines is $(2, -2)$. One of these two lines could be:
A $x - y = 0$
b $2x + 2y = 8$
c $2x + 2y = 0$
d $2x - 2y = 4$
e $2x - 2y = 0$

13 Paul makes rulers. There is a fixed cost of $60 plus a manufacturing cost of $0.20 per ruler. Last week Paul was able to break even by selling his rulers for $1 each.
The number of rulers Paul sold last week was:
A 50  B 75  C 90  D 120  E 150

14 A printing firm charges a fixed amount of $50 plus $10 per 25 invitations. An order for 300 wedding invitations costs (in dollars):
A 120  B 170  C 720  D 800  E 3050
The following information refers to questions **15** and **16**.

The line segment graph below represents a journey taken by an ant.

![Line segment graph](image)

15 The speed of the ant for the interval CD was:
A 8 cm per minute  B 10 cm per minute
C 12 cm per minute  D 14 cm per minute
E 16 cm per minute

16 The average speed of the ant over the entire trip was:
A equal to its speed over the interval AB
B equal to its speed over the interval CD
C less than its speed over the interval CD
D greater than its speed over the interval CD
E greater than its speed over the interval AB

The following graph relates to questions **17** and **18**.

![Graph](image)

A gas-powered camping lamp is lit and the gas is left on for six hours. During this time the lamp runs out of gas.

The graph shows how the mass, $M$, of the gas container (in grams) changes with time, $t$ (in hours), over this period.

17 Assume that the loss in weight of the gas container is due only to the gas being burnt.

From the graph it can be seen that the lamp runs out of gas after:
A 1.5 hours  B 3 hours  C 4.5 hours
D 6 hours  E 220 hours

18 Which one of the following rules could be used to describe the graph?

\[ M = \begin{cases} 332.5 - 25t & \text{for } 0 \leq t \leq 4.5 \\ 220 & \text{for } 4.5 < t \leq 6 \end{cases} \]

A  \[ M = \begin{cases} 332.5 - 25t & \text{for } 0 \leq t \leq 4.5 \\ 220t & \text{for } 4.5 < t \leq 6 \end{cases} \]

B  \[ M = \begin{cases} 332.5 + 25t & \text{for } 0 \leq t \leq 4.5 \\ 220t & \text{for } 4.5 < t \leq 6 \end{cases} \]

C  \[ M = \begin{cases} 332.5 - 12.5t & \text{for } 0 \leq t \leq 4.5 \\ 220 & \text{for } 4.5 < t \leq 6 \end{cases} \]

D  \[ M = \begin{cases} 332.5 - 12.5t & \text{for } 0 \leq t \leq 4.5 \\ 220t & \text{for } 4.5 < t \leq 6 \end{cases} \]

E  \[ M = \begin{cases} 332.5 - 12.5t & \text{for } 0 \leq t \leq 4.5 \\ 220t & \text{for } 4.5 < t \leq 6 \end{cases} \]

19 The line of best fit was drawn on a scatterplot as shown below.

Which of the following statements is not true?

A The independent variable is $m$.
B The line can be used to predict the value of $m$, given $n$.
C The line can be used to predict the value of $n$, given $m$.
D $n$ is the dependent variable.
E The equation of the line could be $n = 100 - 2m$.

**SHORT ANSWER**

1 Plot the graphs of the following linear equations:

\[ a \quad 2x - 4y + 6 = 0 \quad b \quad -4 = 2x + y \]

2 Consider the equation $y = 0.25x - 0.5$.

\[ a \quad \text{Complete the following table of values, using a CAS calculator.} \\
\begin{array}{c|c|c|c|c|c|c}
\hline
x & -3 & -2 & -1 & 0 & 1 & 2 \\
\hline
y & & & & & & \\
\hline
\end{array} \\
\]

\[ b \quad \text{Show the graph of the function for } -3 \leq x \leq 3. \]

3 The line which passes through the points $(2, y)$ and $(5, 9)$ has a gradient of 2.

\[ a \quad \text{Find the value of } y. \]

\[ b \quad \text{Which of the following points lies on this line: } (3, 6), (4, 7) \text{ or } (6, 13)? \]
4. Find the equation of the straight line for the graph shown below.

5. a. Find the equation of the line which passes through (−3, 2) and (1, 6).
   b. Find the equation of the other line which is parallel to the first one and intersects the x-axis at x = 5.

6. Consider the line with the equation \(3x + 2y - 6 = 0\). Write the equation of the straight line which:
   a. has the same gradient, but is shifted 5 units down
   b. has the same y-intercept, but steeper
   c. is parallel to the given line and passes through the origin.

7. Given the equation of a line is \(2x + y - 1 = 0\):
   a. determine the y-intercept
   b. determine the x-intercept
   c. determine the coordinates of the point on the line when \(x = 1\)
   d. determine the gradient of the line
   e. sketch the graph of the line for \(2 \leq x \leq 2\).

8. Given the equations of two lines, \(y = 2x + 5\) and \(x - 4y = -6\),
   a. sketch the graphs of the lines on the same set of axes
   b. use the graph to estimate the coordinates of the point of intersection of the two lines
   c. find the coordinates of the point of intersection of the two lines algebraically.

9. The graph below shows the cost, \(C\), of a towing service.

   a. Copy and complete the following sentence: The cost of towing a car depends on the _______.
   b. Find the equation of the cost.
   c. Explain the meaning of the gradient and y-intercept values in the equation of the cost.

10. In one particular week, Harry began with 50 litres of fuel in the tank of his van. After he had travelled 160 km, there were 30 litres of fuel left. The amount of fuel remaining in the tank of Harry’s van followed a linear trend as shown in the graph below.

   a. Determine the equation of the line shown in the graph above.
   b. How much further will he be able to travel before the tank is empty?
   c. Harry stopped to refuel his van when there were 12 litres of fuel left in the tank. He completely filled the tank in \(3\frac{1}{2}\) minutes when fuel was flowing from the pump at a rate of 18 litres per minute.
   d. How much fuel does the tank hold when it is completely full? Write your answer in litres.

11. When Mera first started a sure-slim weight-loss diet, she was losing one kilogram per week for the first 9 weeks. After that, her rate of weight loss slowed down to half a kilogram per week. Upon reaching her target weight after 21 weeks, Mera stopped dieting.

   a. Construct a line segment graph to illustrate Mera’s weight-loss. (Place time, \(t\), on the
horizontal axis and weight loss, $W$, on the vertical axis.)

b Calculate Mera’s total weight loss over the 21-week period.

c What was Mera’s average weight loss (in kilograms per week) over the entire time of the diet?

12 Two bushwalkers, Malinda and Christos, set out to walk from Fishbone Creek to Snake Gully, a distance of 20 km. They start at the same time and follow the same route.

a Malinda walks at a constant speed of 4 km/h for the entire journey and takes no rest periods. How far does she travel in 1.5 hours?

b The distance walked by Malinda from Fishbone Creek, in kilometres, is given by the equation $D_m = 4t$ for $0 \leq t \leq 5$, where $t$ is the time in hours since she began walking. Draw and label the graph of $D_m$ against $t$ on the set of axes below.

c Let $D_c$ represent the distance walked by Christos. Draw and label the graph of $D_c$ against $t$ on the set of axis in part b.

d Malinda eventually catches up to Christos. How many hours after they start walking does this happen?

EXAM TIP Most students were able to draw a two-segment graph which had to end at (6, 20). Many stopped at $t = 5$, below the point for Malinda. Others went beyond (6, 20). The correct terminal point was required for full marks and the points had to be joined. A method mark was available for a reasonable two-segment graph from (0, 0).

[Assessment report 2003]

13 The following table shows the number of people who attended a Grand Final over a six-year period.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance ('000)</td>
<td>81</td>
<td>88</td>
<td>96</td>
<td>100</td>
<td>98</td>
<td>104</td>
</tr>
</tbody>
</table>

a Construct a scatterplot showing the relationship between the year and the attendance.

b Find the equation of the line of best fit by eye.

c What does the gradient represent?

d Predict the attendance in the seventh year.

EXAM TIP Most students got both marks although many did not terminate their line at (5, 20). In this question, the terminal was disregarded. Some did not join points with a line and lost 1 mark. One mark was given for a reasonable line with positive gradient from (0, 0).

[Assessment report 2003]

EXTENDED RESPONSE

1 To hire a lawn mower from the local service station, you must pay an initial cost of $5 plus an additional charge of $2.25 per hour.

a How much will it cost to hire a lawn mower for 3 hours?

b Construct an equation for the cost, $C$, if you hire the lawn mower for $t$ hours.

c Graph the equation.

d If the final cost was $18.50, how many hours was it hired for?
The service station across the road offers the same lawn mowers for hire at a flat rate of $18 per day.

e Write the equation of the cost if you hire from this station.

f Graph the equation on the same set of axes you used in part c above.

g From the graph or otherwise determine the minimum number of hours that the mower should be hired so that it is cheaper to hire from the second station.

2 Seraphima decided to run a small business from home by supplying a local supermarket and a coffee shop next door with home-made profiteroles. She invested $2500 in a new oven, mixer and other kitchen appliances and utensils. She then estimated that the ingredients and cost of gas, water and electricity would add up to 50 cents per profiterole.
a Write an equation that describes the total cost \( (C) \) of making \( n \) profiteroles. (The equation must reflect both the initial investment and the cost of baking each profiterole.)

b Seraphima decided to set the price of profiteroles at 70 cents each. Write an equation that describes the revenue, \( R \); that is, the total amount that she earns from selling \( n \) profiteroles. (Assume that she will be able to sell all of the profiteroles that she makes.)

c Sketch the two equations, obtained in parts a and b on the same set of axes.

d The point at which the cost of making \( n \) profiteroles is equal to the revenue from selling them is called the ‘break-even’ point. This point indicates the number of profiteroles that need to be baked and sold, so that there is no profit or no loss made. Find the coordinates of the break-even point from the graph. Check your answer, using algebra.

e What is the minimum number of profiteroles that Seraphima needs to bake and sell in order to start making profit?

f Calculate Seraphima’s profit (or loss) from selling:
  i 10 000 profiteroles  
  ii 16 000 profiteroles.

g From experience, Seraphima knows that she can realistically make only 100 profiteroles a day. How long (in days) will it be until she will start making a profit?

h If Seraphima wishes to start making a profit sooner than calculated in g, she needs to increase the price of her profiteroles. Find the price (to the nearest cent) that Seraphima should charge per profiterole so that her business would start making a profit after one month (30 days) of operation.

3 The UXT electricity company has the following rates, effective from January 2005:

| Peak rates per quarter (including GST) | First 500 kWh: 12.85 cents per kWh  
Next 500 kWh: 13.99 cents per kWh  
Over 1000 kWh: 14.81 cents per kWh |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Off peak rates per quarter (including GST)</td>
<td>All amounts: 7.81 cents per kWh</td>
</tr>
</tbody>
</table>

(kWh = Kilowatt hours.)

a Draw a line segment graph to show the cost of electricity per quarter for the electricity consumption up to and including 1500 kWh, charged at peak rates.

b Find the equation for each segment of the graph.

c Use the equations obtained in part b to find the cost (to the nearest cent) of electricity for each of the following amounts used within one quarter:
  i 358 kWh  
  ii 721 kWh  
  iii 1430 kWh.
d If 1200 kWh were consumed, what is the average rate in cents per kilowatt hour?

e If a quarterly cost of electricity in a particular household was $143.08, how much electricity was consumed by this household?

f During one quarter, due to special circumstances, the amount of Naoum’s electricity bill was calculated using off peak rates instead of peak rates. How much did Naoum save, if he used 612 kWh of electricity in that quarter?

4 Malka is a member of a large network of distributors for an international company. She sells health products, including vitamins, minerals, energy drinks, protein shakes and so on. Her monthly commission rates are as follows: 30% of total sales up to and including $2000, 24% of sales above $2000 and up to $3500, and 20% of any amount in excess of $3500.

a Sketch the line segment graph to represent the amount of monthly commission, C, earned by distributing health products to the total value, v, up to and including $5000.

b Find the equations for each segment of the graph.

c What do the gradients of each segment represent?

d Calculate the total commission earned by Malka in one month if, during that month, her level of sales was $4225.

e During February, Malka earned $1010. What was the total of her February sales?

f What was the overall average rate of Malka’s commissions in February? (Express your answer to the nearest percentage.)

5 The Goldsmith family are going on a driving holiday in Western Australia. On the first day, they leave home at 8 am and drive to Watheroo then Geraldton. The distance–time graph below shows their journey to Geraldton.

At 9.30 am the Goldsmiths arrive at Watheroo. They stop for a period of time.

a For how many minutes did they stop at Watheroo?

After leaving Watheroo, the Goldsmiths continue their journey and arrive in Geraldton at 12 pm.

b What distance (in kilometres) do they travel between Watheroo and Geraldton?

c Calculate the Goldsmiths’ average speed (in km/h) when travelling between Watheroo and Geraldton. The Goldsmiths leave Geraldton at 1 pm and drive to Hamelin. They travel at a constant speed of 80 km/h for 3 hours. They do not make any stops.

d On the graph above, draw a line segment representing their journey from Geraldton to Hamelin.

EXAM TIP The question required a line segment that began at 1 pm and a distance of 310 km and ended at 4 pm and a distance of 550 km. Some students stopped their line segment before 4 pm. A positive gradient was required as the vertical axis represented the distance travelled on their holiday. (Assessment report 2007)
The Goldsmiths’ car can use either petrol or gas. The following equation models the fuel usage of petrol, \( P \), in litres per 100 km (L/100 km) when the car is travelling at an average speed of \( s \) km/h.

\[
P = 12 - 0.02s
\]

The line \( P = 12 - 0.02s \) is drawn on the graph below for average speeds up to 110 km/h.

\[\text{Graph showing fuel usage (L/100 km) vs. average speed (km/h)}\]

\[\text{Axes:} \quad \text{Fuel usage (L/100 km)} \quad \text{Average speed (km/h)}\]

\[\text{Points:} \quad (0, 12), (110, 8.4)\]

\[\text{Line:} \quad P = 12 - 0.02s\]

\[\text{Average speeds up to 110 km/h.}\]

e Determine how many litres of petrol the car will use to travel 100 km at an average speed of 60 km/h. Write your answer correct to 1 decimal place.

f On the axes above, draw the line \( G = 15 - 0.06s \) for average speeds up to 110 km/h.

The Goldsmiths’ car travels at an average speed of 85 km/h. It is using gas. Gas costs 80 cents per litre.

h Determine the cost of the gas used to travel 100 km. Write your answer in dollars and cents.

Star Dance Studio introduced free Thursday dance classes for beginners. The table below shows the number of people attending these classes in the first 8 weeks after the introduction.

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance</td>
<td>15</td>
<td>18</td>
<td>20</td>
<td>23</td>
<td>27</td>
<td>31</td>
<td>33</td>
<td>36</td>
</tr>
</tbody>
</table>

a Plot the data from the table, showing attendance on the vertical axis.
b Find the equation of the line of best fit for the data, using a CAS calculator.
c Add the line of best fit to the graph.
d Comment on the closeness of the fit.
e Write a short statement, giving an interpretation of the meaning of the gradient of the line.

f Assuming that the pattern will continue, predict the number of people who will attend the free dance class 12 weeks after the introduction.
Chapter opener

Digital doc
- 10 Quick Questions: Warm up with ten quick questions on linear graphs and modelling. (page 119)

3A Plotting straight line graphs

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- SkillSHEET 3.1: Practise substitution. (page 121)
- Spreadsheet 093: Investigate plotting relations. (page 121)
- SkillSHEET 3.2: Practise recognising linear functions. (page 122)

3B Using a CAS calculator to plot and sketch linear functions

Tutorial
- WE4 int-0863: Watch how to sketch the graph of a linear function using a CAS calculator. (page 123)

Digital doc
- Spreadsheet 041: Investigate graphs using the function grapher. (page 124)

3C Finding the gradient of a straight line

Tutorial
- WE5 int-0864: Watch how to calculate the gradient of a linear graph given two points. (page 125)

Digital docs
- SkillSHEET 3.3: Practise finding the gradient of a straight line. (page 126)
- Spreadsheet 046: Investigate gradients of straight lines. (page 126)

3D Determining the equation of a straight line

Tutorial
- WE6 int-0865: Watch how to find the equation of a straight line given two points. (page 129)

Digital docs
- WorkSHEET 3.1: Plot and sketch, calculate gradients and determine equations of linear graphs. (page 132)
- Investigation: Relate equations and graphs to values of gradients and intercepts. (page 132)

3E Sketching straight line graphs

Tutorial
- WE9 int-0866: Watch how to use the gradient–intercept method to sketch the linear graph. (page 133)
- WE12 int-0999: Watch how to use a CAS calculator to determine the points of intersection of two linear graphs, graphically and algebraically. (page 136)

Digital docs
- SkillSHEET 3.4: Practise using the gradient–intercept method for sketching linear graphs. (page 138)
- Spreadsheet 067: Investigate linear graphs. (page 138)

3F Application of linear modelling

Interactivity
- Linear modelling int-0804: Consolidate your understanding of linear modelling. (page 139)

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- WE16 int-0868: Apply linear modelling techniques to sketch a line segment graph to describe the data. (page 142)

Digital docs
- Spreadsheet 041: Investigate graphs using the function grapher. (page 144)
- WorkSHEET 3.2: Determine rules for linear graphs, x- and y-intercepts and gradients and sketch linear graphs. (page 147)

3G Line of best fit (by eye)

Digital doc
- Spreadsheet 041: Investigate a line of best fit. (page 150)

Chapter review

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- Test Yourself Chapter 3: Take the end-of-chapter test to test your progress. (page 161)

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